***Review of Logic***

*CS 536: Science of Programming; Due Wed Aug 31*

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1. **[8 = 4 \* 2 points]**
   1. **Is *p* ∧ *q* ∨ *r* → ¬*p* → *q* ≡** **((*p* ∧ *q*) ∨ *r*) → ((¬*p*) → *q*)? (Or in ASCII, is**

**p and q or r -> !p -> q == ((p and q) or r) -> ((!p) -> q) ?)**

*Ans: TRUE*

*Reason: by removing unnecessary parenthesis from RHS*

*p ∧ q ∨ r → (¬p → q)*

*as p ∧ q ∨ r* *→ (¬p → q) is of the form p → (q→ r) and it is same as*

*p → q → r as “→” follows right associativity*

* 1. **Is** **(*p* ∨ (¬*q*)) ∧ *r* ∧ (*s* → *p*) ≡ *p* ∨ ¬*q* ∧ *r* ∧ (*s* → *p*) ?**

Ans: FALSE

Reason: By removing unnecessary Parenthesis from LHS

(*p* ∨ ¬*q*) ∧ *r* ∧ (*s* → *p*)

As ∧ has more precedence than ∨ hence above is false

* 1. **Is ∀*x* . *p* ∧ ∀*y* . *q* → *r* ≡** **((∀*x* . *p*) ∧ (∀*y* . *q*)) → *r* ?**

Ans: TRUE

Reason: By removing unnecessary Parenthesis from RHS

∀*x* . *p* ∧ ∀*y* . *q* → *r*

Hence Expression is true

* 1. **Is ¬∃*x* . ∀*y* . *p* ∨ *q* ≡** **¬(∃*x* . (∀*y* . (*p* ∨ *q*))) ?**

Ans: TRUE

Reason: By removing unnecessary Parenthesis from RHS

**¬∃*x* . ∀*y* . *p* ∨ *q***

Hence Expression is true

1. **[6 = 2 \* 3 points] Give the minimal parenthesization of each of the following. I.e., what do you get after removing all redundant parentheses? Hint: It may help to attach subscripts to the parentheses to match up (₁ … )₁ and so on.**
   1. **(¬(*p* → (((¬*q*) ∧ *r*) ∨ ((¬*p* ∨ *r*) ∧ (*q* ∧ *s*)))))**

Ans: ¬*p* → ¬*q* ∧ *r* ∨ (7¬*p* ∨ *r*)7 ∧ *q* ∧ *s*

* 1. **(∀i . (((0 ≤ i) ∧ ( i < m)) → (∃j . (((m ≤ j) ∧ ( j < n)) → (b[i] = b[j]))))).**

Ans: ∀i . 0 ≤ i ∧  i < m → ∃j . m ≤ j ∧  j < n → b[i] = b[j]

**(This says that every element b[i] in the array segment b[0..m-1] is equal to some element b[j] in the array segment b[m..n-1]. Note “b[i] in b[0..m-1]” implies**

**0 ≤ i < m and “b[j] in b[m..n–1]” implies *m* ≤ j < n, so 0 < m < n is implied.)**

1. **[6 = 2 \* 3 points] Give the full parenthesization of each of the following.** 
   1. ***p* ∨ *r* ∨ ¬*s* → ¬*q* ∧ *r* → ¬*p* ↔ ¬*s* → *t***

Ans: (1(2(3*p* ∨ *R)3* ∨ (4¬*s)4)2* → (5(6(7¬*q)7* ∧ *R)6* → (8(9(10¬*p)10* ↔ (11¬*s)11)9* → *t)8)5)1*

* 1. **∃m . 0 ≤ m ∧ m < n ∧ ∀j . 0 ≤  j ∧ j < m → b[0] ≤ b[j] ∧ b[j] ≤ b[m]**

Ans: (1(2(3∃m . (4(50 ≤ m)5 ∧ (6m < n)6)4)3 ∧ (7∀j . (80 ≤  j)8 ∧ (9j < m)9)7)2 → (10(11b[0] ≤ b[j])11 ∧ (12b[j] ≤ b[m])12)10)1

1. **[6 = 3 \* 2 points] For each of the following, answer the question “Is it a** **tautology,** **contradiction, or** **contingency?”**
   1. ***p* ∧ *q* ∨ *r* → *p* ∨ *r***

Ans: tautology

Reason: *p* ∧ *q* ∨ *r* → *p* ∨ *r*

**⇔** *((p* ∧ *q)* ∨ *r)* → (*p* ∨ *r) adding parenthesis*

**⇔**¬*(**(p* ∧ *q)* ∨ *r)* ∨ *p* ∨ *r* Definition of →

**⇔**¬*(p* ∧ *q)* ∧ ¬*r* ∨ *p* ∨ *r* DeMorgan’s Laws

**⇔**¬*p* ∨ ¬*q* ∧ ¬*r* ∨ *p* ∨ *r* adding parenthesis

**⇔**¬*p* ∨ (¬*q* ∧ ¬*r)* ∨ *p* ∨ *r* Commutativity

**⇔**¬*p* ∨ (¬*q* ∧ ¬*r)*  ∨ *r* ∨ *p* Associativity

**⇔** ¬*p* ∨ (¬q ∨ r ) ∧ (¬r ∨ r) ∨ *p* Excluded middle

**⇔** ¬*p* ∨ (¬q ∨ r ) ∧ T ∨ *p* Identity

**⇔** ¬*p* ∨ (¬q ∨ r ) ∨ *p* Commutativity

**⇔** ¬*p* ∨ *p* ∨ (¬q ∨ r ) Excluded middle

**⇔** T∨ (¬q ∨ r ) Domination

**⇔** T

* 1. **((*p* → *q*) → *r*) → (*p* → (*q* → *r*))**

Ans: tautology

Reason: (¬ (*p* → *q*)  ∨ *r*) → ( ¬*p* ∨ (*q* → *r*)) Definition of → twice

**⇔**¬ (¬ (¬*p* ∨ *q*)  ∨ *r*)  ∨ ( ¬*p* ∨ (¬*q* ∨ *r*)) DeMorgan’s Laws

**⇔**¬(p ∧ ¬q ∨ r) ∨ (¬*p* ∨ ¬*q* ∨ *r)* DeMorgan’s Laws

**⇔**¬p ∨ (q ∧ ¬r) ∨ ¬*p* ∨ ¬*q* ∨ *r* Commutativity

**⇔**¬p ∨ (q ∧ ¬r) ∨ *r*  ∨ ¬*p* ∨ ¬*q* Associativity

**⇔**¬p V (q ∨ *r)* ∧ (¬r ∨ *r* ) ∨ ¬*p* ∨ ¬*q* Excluded middle

**⇔**¬p V (q ∨ *r)* ∧ T ∨ ¬*p* ∨ ¬*q* Identity

**⇔**¬p V q ∨ *r* ∨ ¬*p* ∨ ¬*q* Commutativity

**⇔**¬p ∨ *r* ∨ ¬*p* ∨ (¬*q* V q) Excluded middle

**⇔**¬p ∨ *r* ∨ ¬*p* ∨ T Domination

**⇔***T*

* 1. **¬(*p* → *q*) ∧  (¬*p* ∨ *q*)**

Ans: contradiction

Reason: ¬(¬*p* ∨ *q*) ∧  (¬*p* ∨ *q*) DeMorgan’s Laws

**⇔***p* ∧ ¬*q* ∧  (¬*p* ∨ *q*) Associativity

**⇔***p* ∧ (¬*q* ∧ ¬*p)* ∨ *(*¬*q* ∧ q) Contradiction

**⇔** *(p* ∧ ¬*q* ∧ ¬*p)* ∨ *F* Commutativity

**⇔** *(p* ∧ ¬*p* ∧ ¬*q)* ∨ *F* Contradiction

**⇔** (F ∧ ¬*q) V F* Domination

**⇔***F V F* Idempotentcy

**⇔***F*

1. **[2 points] First, please recall that ℕ = {0, 1, 2, …} is the set of natural numbers. Consider the predicate (∀*x* ∈ ℕ . *x* \* *y* ≥ *x*), where *y* ∈ ℕ.** 
   1. **Give a value for *y* that makes the predicate false;**

Ans: y<1 which means y=0

* 1. **Give a value for *y* that makes the predicate true.**

Ans: y**≥**1 which means y=1

1. **[6 = 2 \* 3 points] Let *p* be a predicate over *x* and *y* and consider the two predicates**

**(****∀*x* ∈ ℤ . ∃*y*****∈ ℤ . *p*) and** **(∃*y* ∈ ℤ . ∀*x* ∈ ℤ . *p*)**

**(Please recall that ℤ = {…, –2, –1, 0, 1, 2, …} is the set of integers numbers.) If *p* is a tautology like *x*+*y* = *y*+*x* then both predicates are certainly true; similarly, if *p* is a contradiction like *x*+*y* ≠ *y*+*x*, then both predicates are false.**

* 1. **Give an example of a *p* that is a** **contingency and makes both predicates true.**

Ans: p should be a contingency like x\*y = y ie when we have y=0 first and second both will be true

* 1. **Give an example of a *p* that is a contingency and makes the first predicates true but the second predicate false. (In other words, find a *p* such that**

**(∀*x* ∈ ℤ . ∃*y* ∈ ℤ . *p*) → (∃*y* ∈ ℤ . ∀*x* ∈ ℤ. *p*) is false (because** **true → false is false).**

Ans: p should be a contingency like x2=y. Hence for all x we can take some y such that x2=y is true where as there are some y for which there arn’t any x to satify x2=y hence true → false is false

1. **[8 points] Simplify** **¬(∃*x* . (∀*y* . *x* ≤ *y*) ∨ ∀z . *x* ≥ z) to a predicate that has no uses of ¬. (You'll need DeMorgan's laws.) Here is an example of the format to use:**

**¬(*x* < *y*****∧ *y* ≤ z)**

**⇔ ¬(*x* < *y*) ∨ ¬(*y* ≤ z) DeMorgan’s Law ⇔ *x* ≥ *y* ∨ *y* > z** **Negation of comparison (twice)**

**(Don't forget to include the names of the rules you're applying!)**

Ans: ¬(∃*x*.(∀*y*.*x* ≤ *y*) ∨ ∀z . *x* ≥ z) DeMorgan’s Laws Extended

**⇔** ∀*x* . ¬ ((∀*y* . *x* ≤ *y*) ∨ (∀z . *x* ≥ z))  DeMorgan’s Laws

**⇔** ∀*x* .(¬ (∀*y* . *x* ≤ *y*) ∧ ¬ (∀z . *x* ≥ z)) DeMorgan’s Laws Extended (twice)

**⇔** ∀*x* .( ∃*y* . ¬(*x* ≤ *y)* ∧ ¬ ∃z . ¬(*x* ≥ z)) Negation of comparison (twice)

**⇔** ∀*x* .∃*y* . *x* > *y* ∧ ¬ ∃z . *x* < z

1. **[8 points] Repeat the previous problem on** **¬((¬∀*x* . *f*(*x*) ≥ 0) → (∀*x* . *f*(*x*) < 0)).**

**(Again, don't forget to include the names of the rules you're applying.)**

Ans: ¬((¬∀*x* . *f*(*x*) ≥ 0) → (∀*x* . *f*(*x*) < 0)) Definition of →

**⇔** ¬(¬ (¬∀*x* . *f*(*x*) ≥ 0) V (∀*x* . *f*(*x*) < 0)) DeMorgan’s Laws Extended

**⇔** ¬((¬¬∀*x* .¬ *f*(*x*) ≥ 0) V (∀*x* . *f*(*x*) < 0)) Double negation

**⇔** ¬((∀*x* .¬ *f*(*x*) ≥ 0) V (∀*x* . *f*(*x*) < 0)) Negation of comparison

**⇔** ¬((∀*x* . *f*(*x*) < 0) V (∀*x* . *f*(*x*) < 0)) DeMorgan’s Laws

**⇔** (¬ (∀*x* . *f*(*x*) < 0)  ∧  ¬ (∀*x* . *f*(*x*) < 0)) DeMorgan’s Laws Extended (twice)

**⇔(**∃*x* . ¬*f*(*x*) < 0) ∧ (∃*x*  .¬ *f*(*x*) < 0) Negation of comparison (twice)

**⇔** ∃*x* . *f*(*x*)  ≥  0 ∧ ¬ ∃*x* . *f*(*x*)  ≥ 0